

## Exercise 57

In this exercise we estimate the rate at which the total personal income is rising in the Richmond-Petersburg, Virginia, metro-politan area. In 1999, the population of this area was 961,400, and the population was increasing at roughly 9200 people per year. The average annual income was \$30,593 per capita, and this average was increasing at about \$1400 per year (a little above the national average of about \$1225 yearly). Use the Product Rule and these figures to estimate the rate at which total personal income was rising in the Richmond-Petersburg area in 1999. Explain the meaning of each term in the Product Rule.

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### Solution

Let  $P(t)$  be the population  $t$  years after 1999. Then

$$P(0) = 961\,400$$

and

$$P'(0) = 9200.$$

Let  $A(t)$  be the average annual income per capita  $t$  years after 1999. Then

$$A(0) = 30\,593$$

and

$$A'(0) = 1400.$$

The total personal income is given by the product of  $P(t)$  and  $A(t)$ .

$$I(t) = P(t)A(t)$$

The rate at which the total personal income rises is given by the derivative.

$$I'(t) = \frac{d}{dt}[P(t)A(t)] = P'(t)A(t) + P(t)A'(t)$$

Note that  $P'(t)$  represents the rate at which the population changes with respect to time at time  $t$ , and  $A'(t)$  is the rate that the average annual income per capita changes with respect to time at time  $t$ . Evaluate  $I(t)$  at  $t = 0$  to get the rate that personal income increases in 1999.

$$I'(0) = P'(0)A(0) + P(0)A'(0) = (9200)(30\,593) + (961\,400)(1400) = \$1\,627\,415\,600$$